The CENTRE for IN-SCHOOL COMPETITIONS
NACIS MATH DEPARTMENT

Möbius Infinity Contest

G10

Tuesday, January 14 (75 minutes)

Total Time: 75 minutes

Do not open this booklet until instructed to do so.

Number of questions: *8*

No calculators or other electronic devices are allowed.

- Students may not use any form of calculator (graphing, scientific, or otherwise).
- All personal belongings and electronic devices must be stored unless otherwise authorized by the supervisor.

Parts of each question can be of two types:

1. **SHORT ANSWER (S)**
   - Worth 1 or 2 marks each
   - Full marks are awarded for a correct answer placed in the box
   - Part marks are awarded only if relevant work is shown
2. **FULL SOLUTION (W)**
   - Worth 7 marks each
   - Must be written in the appropriate location in the answer booklet
   - Marks are awarded for completeness, clarity, and style of presentation
   - A correct solution poorly presented will not earn full marks

WRITE ALL ANSWERS IN THE ANSWER BOOKLET PROVIDED.

- Express answers as simplified exact numbers where possible (e.g., $\pi + 1$, $1 - \sqrt{2}$)

Problem 1 (S):

Given the equation

$$\frac{17}{10} = 1 + \frac{1}{a + \frac{1}{b + \frac{1}{c}}},$$

where a, b, and c are positive integers, determine the values of a, b, and c.

Problem 2 (S): Eight people, including triplets Barry, Carrie and Mary, are going for a trip in four canoes. Each canoe seats two people. The eight people are to be randomly assigned to the four canoes in pairs. What is the probability that no two of Barry, Carrie and Mary will be in the same canoe?

Problem 3 (W):

What is the value of

$$\left(\sum_{k=1}^{200} \log_{12^k}\left(13^{k^2}\right)\right) \cdot \left(\sum_{k=1}^{10} \log_{169^k}\left(144^k\right)\right)?$$

Problem 4 (W):

A function f satisfies the equation

$$f(x) + f\left(\frac{1}{1-x}\right) = 24x \quad \text{for all real } x \text{ except } x = 0 \text{ and } x = 1.$$

Determine the value of $f(3)$.

Problem 5 (W): Drew has 4 identical black socks, 4 identical red socks, 2 identical white socks, and 2 identical grey socks in his drawer. He selects 5 socks at random (all at once) from the drawer.

The probability that he obtains *exactly one* matching pair of socks (and does **not** obtain a second matching pair) is $\frac{m}{n}$,where m and n are positive integers with no common factor other than 1.Find $m + n^2$.

Problem 6 (S):

Let x and y be positive integers such that

$$\frac{1}{x} + \frac{1}{2y} = \frac{1}{40}.$$

What is the maximum possible value of y?

Question 7 (W):

Consider the function

$$f(x) \; = \; x^2 \; - \; 2x.$$

Determine all real numbers x such that

$$f\big(f\big(f(x)\big)\big) \; = \; 3.$$

Question 8 (W): Given a regular polygon with n sides **circumscribed outside** a circle of radius r, denoted as regular n-gon $P_1P_2P_3 \ldots P_n$. Let the starting point P_1 be on the y-axis and let the vertices be arranged counterclockwise.

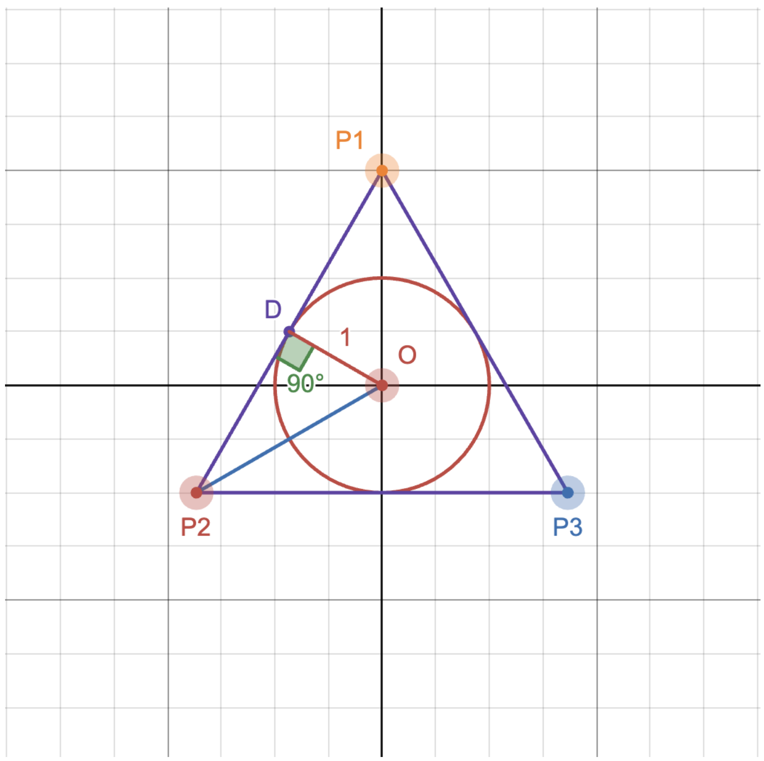

Figure 1: Question 6(a)

(a) Assume $\mathbf{r = 1}$, calculate the coordinates of the vertices P_2 and P_3 for a regular triangle ($n = 3$), keeping the expressions in terms of radicals. **(S)**

(b) Derive the general formula for the coordinates of vertex P_k ($k = 1, 2, \ldots, n$) in terms of n and r, keeping expressions in terms of radicals and trigonometric functions. **(W)**