The CENTRE for IN-SCHOOL COMPETITIONS
NACIS MATH DEPARTMENT

Möbius Infinity Contest

G6

Tuesday, January 14 (75 minutes)

Total Time: 75 minutes

Do not open this booklet until instructed to do so.

Number of questions: *9*

No calculators or other electronic devices are allowed.

- Students may not use any form of calculator (graphing, scientific, or otherwise).
- All personal belongings and electronic devices must be stored unless otherwise authorized by the supervisor.

Parts of each question can be of two types:

1. **SHORT ANSWER (S)**
   - Worth 2 or 3 marks each
   - Full marks are awarded for a correct answer placed in the box
   - Part marks are awarded only if relevant work is shown
2. **FULL SOLUTION (W)**
   - Worth the remainder of the 10 marks for the question
   - Must be written in the appropriate location in the answer booklet
   - Marks are awarded for completeness, clarity, and style of presentation
   - A correct solution poorly presented will not earn full marks

WRITE ALL ANSWERS IN THE ANSWER BOOKLET PROVIDED.

- Express answers as simplified exact numbers where possible (e.g., $\pi + 1$, $1 - \sqrt{2}$)

Question 1 (S):

If $x = 3$, what is the value of

$$\frac{x^4 - 5x^2}{x^2} ?$$

如果 $x = 3$，那么请计算该表达的式的值

$$\frac{x^4 - 5x^2}{x^2} ?$$

Question 2 (S):
Compute the value of

$$\left[13 \div \left(\tfrac{1}{11} + \frac{2\frac{1}{4}}{1 - \frac{1}{100}}\right) - 1 \div 7\right] \div 1\tfrac{11}{14} =$$

计算下列表达式的值

$$\left[13 \div \left(\tfrac{1}{11} + \frac{2\frac{1}{4}}{1 - \frac{1}{100}}\right) - 1 \div 7\right] \div 1\tfrac{11}{14} =$$

Question 3 (W):

对于下列不等式，存在多个自然数 n 使得不等式成立。设所有满足条件的自然数之和为 x。
请问，x 等于多少?

$$\frac{2}{7} \div \left(\tfrac{n}{8}\right) > \frac{1}{6}.$$

Question 4 (S): When Xiaohong was organizing her coin purse, she found that there were 25 coins in total of denominations 1 fen, 2 fen, and 5 fen, adding up to 0.60 yuan. What is the maximum possible number of 5-fen coins?

小红整理零钱包时发现，包中有面值为 1 分、2 分、5 分的硬币共 25 枚，总值为 0.60 元。则 5 分的硬币最多有多少枚?

Question 5 (W):
Define the operation "$\oplus$" as follows:

$$a \oplus b = \begin{cases} a & (\text{if } a > b), \\ 1 & (\text{if } a = b), \\ b & (\text{if } a < b). \end{cases}$$

For example:

$$3.5 \oplus 2 = 3.5, \quad 1 \oplus 1.2 = 1.2, \quad 7 \oplus 7 = 1.$$

Then compute:

$$\frac{1.1 \oplus \left(\frac{7}{3} - \frac{1}{3}\right) \oplus 0.1}{\frac{4}{5} \oplus 0.8} = ?$$

定义运算"$\oplus$":

$$a \oplus b = \begin{cases} a & (\text{若} a > b), \\ 1 & (\text{若} a = b), \\ b & (\text{若} a < b). \end{cases}$$

例如:

$$3.5 \oplus 2 = 3.5, \quad 1 \oplus 1.2 = 1.2, \quad 7 \oplus 7 = 1.$$

则

$$\frac{1.1 \oplus \left(\frac{7}{3} - \frac{1}{3}\right) \oplus 0.1}{\frac{4}{5} \oplus 0.8} = ?$$

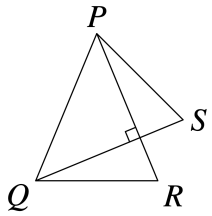
Question 6 (W): Boxes A, B, C, and D each contain some balls. A portion of the balls in box A is transferred to the other three boxes as follows: For every ball originally in box A, that many additional balls are placed into the other boxes. Then, using the same procedure, the balls in boxes B, C, and D are each transferred in turn. In the end, all four boxes have 16 balls. Which box originally contained the most balls, and how many did it contain?
A, B, C, D 四个箱子中分别装有一些小球，现将 A 箱中的部分小球按如下要求转移到其他三个箱子中：该箱中原有几个小球，就再放入几个小球。此后，按照同样的方法依次把 B、C、D 箱中的小球转移到其他箱子中。此时，四个箱子中都有 16 个小球，那么开始时装有小球最多的是哪个箱子，其中装有小球多少个。

Question 7 (W):

In the following addition, x and y represent different single digits. Given that the result is $1xx7$, find the value of $x + y$.
在下面的加法算式中，x 和 y 表示不同的一位数字。已知计算结果为 $1\,x\,x\,7$，求 $x + y$ 的值。

$$\begin{array}{r} 7\,7\,x \\ 6\,y\,x \\ +\,y\,y\,x \\ \hline 1\,x\,x\,7 \end{array}$$

Question 8 (W):



In the diagram shown, $PQ = PR = QS$. Line segments PR and QS are perpendicular to each other. What is the sum of $\angle PRQ$ and $\angle PSQ$?

如图所示，已知 $PQ = PR = QS$，且线段 PR 与 QS 互相垂直。试求 $\angle PRQ$ 与 $\angle PSQ$ 的和。

Question 9 (W): The digits from 1 to 9 are written in ascending order so that each digit n is repeated n times. This creates the block of digits:

$$1\,2\,2\,3\,3\,3\,4\,4\,4\,4 \ \ldots \ 9\,9\,9\,9\,9\,9\,9\,9\,9.$$

This entire block is then repeated **2025** times, producing a long string of digits.

Which digit occupies the 23456th position in this long string?

将数字 1 到 9 按升序排列，并令每个数字 n 重复出现 n 次，即可得到下列数字序列：

$$1 \quad 22 \quad 333 \quad 4444 \quad \ldots \quad 999999999.$$

然后将上述整段序列重复 2025 遍，从而形成一个极长的数字串。请问：在这个长数字串中，第 23456 个位置上的数字是多少?