The CENTRE for IN-SCHOOL COMPETITIONS
NACIS MATH DEPARTMENT

Möbius Infinity Contest

G7 (PART B)

Part A: Monday, January 13 (35 minutes)
Part B: Tuesday, January 14 (35 minutes)

Total Time: 35 minutes

Do not open this booklet until instructed to do so.

Number of questions: *4*
No calculators or other electronic devices are allowed.

- Students may not use any form of calculator (graphing, scientific, or otherwise).
- All personal belongings and electronic devices must be stored away unless otherwise permitted by the supervisor.

Parts of each question can be of two types:

1. **SHORT ANSWER (S)**
   - Worth 2 or 3 marks each
   - Full marks are awarded for a correct answer placed in the box
   - Part marks are awarded only if relevant work is shown
2. **FULL SOLUTION (W)**
   - Worth the remainder of the 10 marks for the question
   - Must be written in the appropriate location in the answer booklet
   - Marks are awarded for completeness, clarity, and style of presentation
   - A correct solution poorly presented will not earn full marks

WRITE ALL ANSWERS IN THE ANSWER BOOKLET PROVIDED.

- Express answers as simplified exact numbers where possible (e.g., $\pi + 1$, $1 - \sqrt{2}$)

Question 1 (S):

A rectangle has its length and width in the ratio 3 : 2. If its perimeter is 50 cm, find the area of the rectangle.

Question 2 (S):

The number 2025 is a perfect square.

(a) Write 2025 as a product of its prime factors.

(b) If 2025 is the area of a square (in cm^2), find the perimeter of this square.

Question 3 (W):

Consider all 3-digit positive integers whose digits have a product of 24. Let X be the total number of such integers. Compute the value of

$$X + \sqrt{2025}.$$

Question 4 (W):

A three-digit integer is any integer from 100 to 999, inclusive. We call a three-digit integer *Locked* if there is no rearrangement of its digits that forms a *smaller* three-digit integer.

For example,

$$138, \quad 309, \quad \text{and} \quad 566$$

are Locked because in each case, any valid three-digit rearrangement of its digits is *not* smaller. However,

$$452, \quad 360, \quad \text{and} \quad 727$$

are *not* Locked, because in each case there exists a three-digit rearrangement that is smaller.

How many three-digit integers are Locked?

(Provide a well-reasoned, clearly organized solution.)

Question 5 (W):

Suppose there are four prime numbers p, q, r, s such that all of the following eight numbers are distinct primes:

$$p, \quad q, \quad r, \quad s, \quad (p+q), \quad (q+r), \quad (260+s), \quad (260-s).$$

Additionally, we know that

$$p + q + r = 260,$$

and both r and s are less than 60.

Either find the value of (p - r) if such primes exist, or show that no such set of primes can exist.

(Provide a clear and complete solution, justifying all steps.)

Question 6 (S):

The digits from 1 to 9 are written in ascending order so that each digit n is repeated n times. This creates the block of digits:

$$1\,22\,333\,4444\ \ldots\ 999999999.$$

This entire block is then repeated **2025** times, producing a long string of digits.

Which digit occupies the 23456th position in this long string?

将数字 1 到 9 按升序排列，并令每个数字 n 重复出现 n 次，即可得到下列数字序列：

$$1 \quad 22 \quad 333 \quad 4444 \quad \ldots \quad 999999999.$$

然后将上述整段序列重复 2025 遍，从而形成一个极长的数字串。请问：在这个长数字串中，第 23456 个位置上的数字是多少?

Problem 7 (W):

Solve the equation in real numbers:
解以下关于实数 x 的方程

$$(5-x)^{2x^2-10x+12} = 1.$$

Problem 8 (W):

An isosceles triangle has base 18 and height 15. The apex is directly above the midpoint of the base. A semicircle is drawn so that its diameter is contained entirely within the base, and the curved arc is tangent to the two equal sides. Find the radius of this semicircle.

(Show all steps in your work. You may draw a diagram to aid in understanding.)

在一等腰三角形中，底边长度为 18，高为 15，且该三角形的顶点正好位于底边中点正上方。在底边上作一条半圆，其直径完全包含在底边之内，且半圆弧与该三角形的两条相等边相切。求该半圆的半径。

(请写出完整的解题步骤，可适当画图帮助理解。)

Problem 9 (W):

A bag contains exactly four balls, each of which is either red or white. One ball is drawn at random from the bag and set aside (not replaced). Then a second ball is drawn at random. Given that the probability of drawing two red balls in succession is$\frac{1}{2}$,determine the probability that both drawn balls are white.

一个袋子里恰好有 4 个球，每个球都是红球或白球。先从袋中随机抽取一个球并取出（不放回），然后再次随机抽取一个球。已知依次抽到两颗红球的概率为 $\frac{1}{2}$，求依次抽到两颗白球的概率。