The CENTRE for IN-SCHOOL COMPETITIONS
NACIS MATH DEPARTMENT

ACME

Möbius Infinity Contest

G8

Tuesday, January 14 (75 minutes)

Total Time: 75 minutes

Do not open this booklet until instructed to do so.

Number of questions: *7*
No calculators or other electronic devices are allowed.

- Students may not use any form of calculator (graphing, scientific, or otherwise).
- All personal belongings and electronic devices must be stored unless otherwise authorized by the supervisor.

Parts of each question can be of two types:

1. **SHORT ANSWER (S)**
   - Worth 2 or 3 marks each
   - Full marks are awarded for a correct answer placed in the box
   - Part marks are awarded only if relevant work is shown
2. **FULL SOLUTION (W)**
   - Worth the remainder of the 10 marks for the question
   - Must be written in the appropriate location in the answer booklet
   - Marks are awarded for completeness, clarity, and style of presentation
   - A correct solution poorly presented will not earn full marks

WRITE ALL ANSWERS IN THE ANSWER BOOKLET PROVIDED.

- Express answers as simplified exact numbers where possible (e.g., $\pi + 1$, $1 - \sqrt{2}$)

Question 1 (S):

The integer 2025 can be written both as a perfect square and as the sum of consecutive cubes of certain natural numbers.

(a) Find those consecutive natural numbers and show that the sum of their cubes equals 2025.

(b) If 2025 is the area of a square (in cm^2), determine the perimeter of this square.

Problem 2 (s):

A box contains 4 white marbles and 6 black marbles. Two marbles are drawn from the box *one after the other*, without replacement. What is the probability that the *second* marble drawn is white?

Problem 3 (W):

A right triangle has its three side lengths (from smallest to largest) given by

$$x, \quad x^2, \quad \text{and} \quad x^3,$$

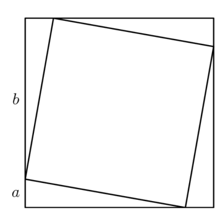
where x is a positive real number. Assuming the side of length x^3 is the hypotenuse, determine the exact value of x.

Question 4 (W): An isosceles triangle has base 18 and height 15. The apex is directly above the midpoint of the base. A semicircle is drawn so that its diameter is contained entirely within the base, and the curved arc is tangent to the two equal sides. Find the radius of this semicircle.

Problem 5 (W): Solve the equation in real numbers:

$$(5-x)^{2x^2-4x+1} = 1.$$

Problem 6 (W): A square with area 16 is inscribed in a square with area 25, with each vertex of the smaller square on a side of the larger square. A vertex of the smaller square divides a side of the larger square into two segments, one of length a, and the other of length b. What is the value of $a+b$?



Question 7:

A new operation "$\star$" on the real numbers is defined by

$$a \star b \;=\; a^2 \;+\; ab \;+\; b^2 \;+\; a \;+\; b \;+\; 7.$$

Use this definition to answer the following:

(a) **Compute:** $(2 \star 3) \star 4$ **(S)**

(b) **Solve an equation:**

$$x \star 5 \;=\; 128.$$

Find all real value(s) of x **(S)**

(c) **Check commutativity.(W)**

- Investigate whether $a \star b = b \star a$ for all real a and b.
- If the operation is commutative, prove it by direct algebraic simplification. If it fails, give a specific pair (a, b) for which the two sides differ.

(d) **Identity element?(W)**

- An identity e would satisfy $a \star e = a$ **and** $e \star a = a$ for all a.
- Check if such an e exists. If yes, find it and show it works in the formula. If no, explain why none can satisfy both conditions.

(e) **Associativity check.(W)**

$$(\,a \star b\,) \star c \;\stackrel{?}{=}\; a \star (\,b \star c\,).$$

If you suspect the operation is *not* associative, find one clear example (a, b, c) for which the two sides differ, and show the numeric values to confirm the failure.