The CENTRE for IN-SCHOOL COMPETITIONS
NACIS MATH DEPARTMENT

Möbius Infinity Contest

G9

Tuesday, January 14 (75 minutes)

Total Time: 75 minutes

Do not open this booklet until instructed to do so.

Number of questions: *8*

No calculators or other electronic devices are allowed.

- Students may not use any form of calculator (graphing, scientific, or otherwise).
- All personal belongings and electronic devices must be stored unless otherwise authorized by the supervisor.

Parts of each question can be of two types:

1. **SHORT ANSWER (S)**
   - Worth 2 or 3 marks each
   - Full marks are awarded for a correct answer placed in the box
   - Part marks are awarded only if relevant work is shown
2. **FULL SOLUTION (W)**
   - Worth the remainder of the 10 marks for the question
   - Must be written in the appropriate location in the answer booklet
   - Marks are awarded for completeness, clarity, and style of presentation
   - A correct solution poorly presented will not earn full marks

WRITE ALL ANSWERS IN THE ANSWER BOOKLET PROVIDED.

- Express answers as simplified exact numbers where possible (e.g., $\pi + 1$, $1 - \sqrt{2}$)

Question 1 (S):

If $x = 3$, what is the value of

$$\frac{x^4 \ - \ 5x^2}{x^2} \ ?$$

Question 2 (S):

A bag contains exactly four balls, each of which is either red or white. One ball is drawn at random from the bag and set aside (not replaced). Then a second ball is drawn at random. Given that the probability of drawing two red balls in succession is$\frac{1}{2}$,determine the probability that both drawn balls are white.

Problem 3 (W):

Solve the equation in real numbers:

$$(5 - x)^{2x^2 - 4x + 1} \ = \ 1.$$

Provide a complete, clearly reasoned solution, justifying each case you consider.

Question 4 (W):

A function f on the positive integers is defined by the following rules:

- $f(1) = 5$.
- For every positive integer n, $f(2n) = f(n) + 4$.
- For every positive integer m, $f(2m + 1) = 2\,f(m)$.

Determine the value of $f(3) + f(4) + f(5)$.

Question 5 (W):

In a right triangle ABC with a right angle at B, suppose the side AB measures 10 units, the side BC measures $t - 1$ units, and the side AC measures $t + 1$ units. Determine the numerical value of t.

Question 6 (W): A medical test is used to screen for a certain disease in a population. The disease prevalence is 5%, meaning 5% of the entire population actually has the disease. The test has a **90% chance** of returning a positive result if a person truly has the disease (i.e., the *sensitivity* is 90%). However, when a person does *not* have the disease, the test still returns a positive result 10% of the time (i.e., the *false positive* rate is 10%).

If a randomly selected individual from the population is tested and the test result is positive, what is the probability that this person *actually* has the disease?

Question 7 (W):

Consider the function

$$f(x) = x^2 - 2x.$$

Determine all real numbers x such that

$$f\big(f\big(f(x)\big)\big) = 3.$$

Question 8 (W): Given a regular polygon with n sides **circumscribed outside** a circle of radius r, denoted as regular n-gon $P_1P_2P_3 \ldots P_n$. Let the starting point P_1 be on the y-axis and let the vertices be arranged counterclockwise.

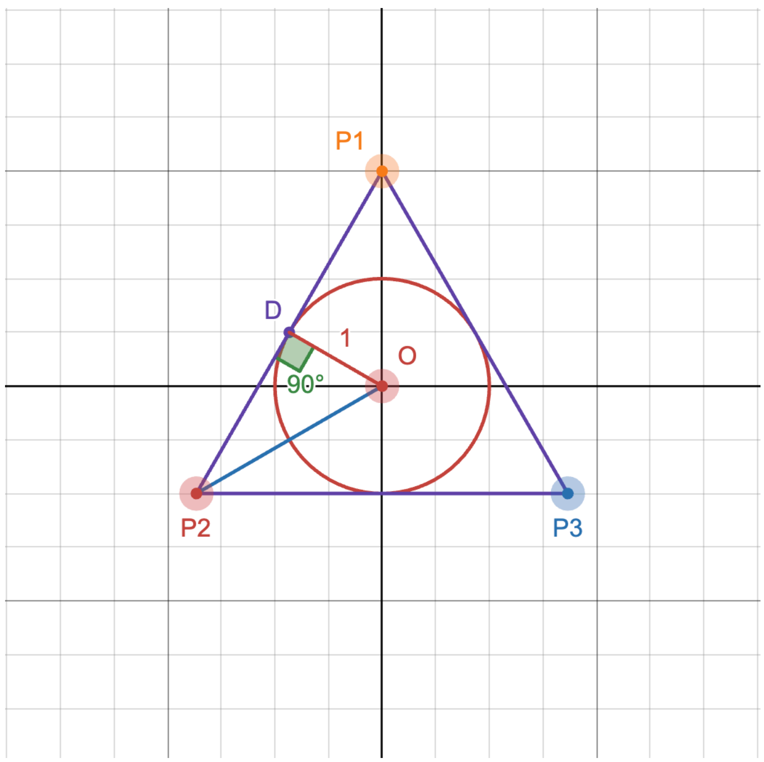

Figure 1: Question 6(a)

(a) Assume $\mathbf{r = 1}$, calculate the coordinates of the vertices P_2 and P_3 for a regular triangle ($n = 3$), keeping the expressions in terms of radicals. **(S)**

(b) Derive the general formula for the coordinates of vertex P_k ($k = 1, 2, \ldots, n$) in terms of n and r, keeping expressions in terms of radicals and trigonometric functions. **(W)**